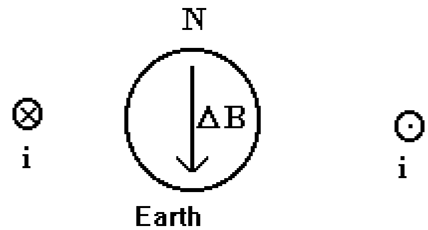
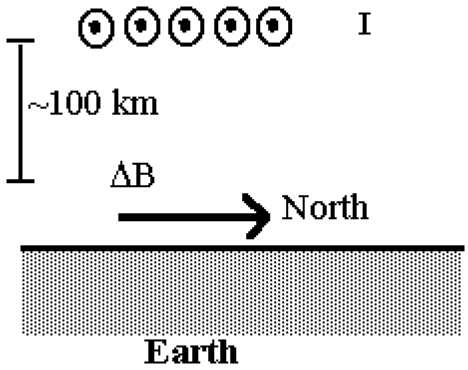
4.1 The activity index, Dst, can reach ~100 nT. Estimate the current necessary to produce this magnetic field effect. Assume the current is a "ring" around the earth, at 3 R_E. That is, if $\Delta B \sim 100$ nT, use the Biot-Savart law for a loop to estimate the current, i.. To keep life simple, assume that the standard formula (e.g. Halliday and Resnick) for the field at the center of a ring is reasonably close to the formula for the field off the axis (e.g. the surface of the earth).



Following Halliday and Resnick (or similar), the magnetic field at the center of a current loop of radius "R" is: $B = \dfrac{\mu_o\, i}{2\,R}$. Inverting the formula to solve for the current, and plugging in the numbers, we get:

$$i = \frac{2\,B\,R}{\mu_o} = \frac{2 \bullet \left(100 \times 10^{-9}\right) \bullet \left(3 \bullet 6.37 \times 10^{6}\right)}{4\pi \times 10^{-7}} = \frac{3.822}{4\pi \times 10^{-7}} = 3.04 \times 10^{6} \text{ Amperes}$$

4.2 The diurnal current variations are due to currents flowing in the upper atmosphere (ionosphere). Take this current to be an infinite, planar (e.g. sheet) current. How large a current density is needed to produce a 100 nT perturbation (ΔB)?



This is an Ampere's Law problem – $\oint \mathrm{B} \bullet \mathrm{d}l = \mu_o i = \mu_o \iint J \bullet dS$

$$B \bullet 2L = \mu_o \, \mathrm{J} \bullet \mathrm{L} \bullet \mathrm{t}$$

where L is the length of the region (in the north direction), J is the current density, t is the thickness of the current sheet. (about 10 km).

$$J = \frac{B \bullet 2L}{\mu_o \bullet \mathrm{L} \bullet \mathrm{t}} = \frac{2B}{\mu_o \, \mathrm{t}} = \frac{200 \times 10^{-9}}{4\pi \times 10^{-7} \bullet 10^4} = 1.59 \times 10^{-5} \ \frac{\mathrm{Amperes}}{\mathrm{m}^2}$$

which may not seem like a lot – but over a cross-sectional area of 1-100 million square meters – it is a lot of amperes.

Probably a better problem to solve is to treat the current just above the surface of the earth as a long wire - $\mathrm{B} = \frac{\mu_o \, \mathrm{i}}{2 \pi \mathrm{R}}$, again solving for the current –

$$\mathrm{i} = \frac{2\pi \mathrm{B} \, \mathrm{R}}{\mu_o} = \frac{2\pi \bullet \left(100 \times 10^{-9}\right) \bullet \left(100 \times 10^{3}\right)}{4\pi \times 10^{-7}} = \frac{0.02\pi}{4\pi \times 10^{-7}} = 50 \times 10^3 \ \mathrm{Amperes}$$